Perihelion precession and deflection of light in gravitational field of a wormhole

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Wormholes

$$ds^2 = e^{2\phi(r)}dt^2 - \frac{dr^2}{1 - \frac{b(r)}{r}} - r^2 d\Omega^2,$$

- general form of a metrics with spherical symmetry

$$b(r_0) = r_0, \quad b'(r_0) < 1.$$

- confition of wormhole existence;

$$l(r) = \pm \int_{r_0}^{r} \frac{dr'}{1 - \frac{b(r')}{r'}},$$

- new coordinate;

Metrics

$$ds^2 = \left(1 - \frac{r_h}{r}\right)^{2+2\delta} c^2 dt^2 - \frac{dr^2}{1 - \frac{r_h}{r}\left[1 + \left(1 - \frac{r_h}{r}\right)^{1-\delta}\right]} - r^2 d\Omega^2.$$

Delta is assumed to be very small with respect to unity.

(Kardashev, Novikov, Shatskiy, A. Reports, 50, 601, (2006), astro-ph/0610441)

Equation of state

$$1 + \delta = -p_{\|}/\varepsilon = p_{\perp}/\varepsilon,$$

Hamilton-Jacobi equation

$$g^{ik}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^k}=m^2c^2$$

$$\left(1-\frac{r_h}{r}\right)^{-2-2\delta}\left(\frac{\partial S}{c\partial t}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial\varphi}\right)^2-$$
$$-\left(1-\frac{r_h}{r}\right)\left[1-\frac{r_h}{r}\left(1-\frac{r_h}{r}\right)^{-\delta}\right]\left(\frac{\partial S}{\partial r}\right)^2=m^2c^2.$$

Its solution is

$$S=-Et+L\varphi+$$
$$+\int\sqrt{\frac{\left(1-\frac{r_h}{r}\right)^{-2-2\delta}\frac{E^2}{c^2}-\frac{L^2}{r^2}-m^2c^2}{\left(1-\frac{r_h}{r}\right)\left[1-\frac{r_h}{r}\left(1-\frac{r_h}{r}\right)^{-\delta}\right]}}\,dr.$$

Equations of motion

$$
\begin{aligned}
m\frac{dt}{ds} &= \frac{E}{c^2\left(1-\frac{r_h}{r}\right)^{2+2\delta}}, \\
m\frac{dr}{ds} &= \left[\frac{E^2}{c^2\left(1-\frac{r_h}{r}\right)^{2+2\delta}} - \frac{L^2}{r^2} - m^2c^2\right]^{1/2} \times \\
&\quad \times \left[1-\frac{r_h}{r}\left(1+\left(1-\frac{r_h}{r}\right)^{1-\delta}\right)\right]^{1/2}, \\
m\frac{d\varphi}{ds} &= \frac{L}{r^2}.
\end{aligned}
$$

We consider a wormhole with spherical symmetry, so the fourth equation is not necessary (we consider the motion in “equatorial” plane only).

Perihelion precession

$$r = r' + r_h$$

We introduce new variable r'

$$\begin{aligned} S &= -Et + L\varphi + \int \left[\left(\frac{E^2}{c^2} + 2Em \right) + \right. \\ &+ 2\left[2Em\,(2+\delta) + m^2c^2\,(1+\delta)\right] \frac{r_h}{r} - \\ &- \left. \frac{1}{r^2}\left(L^2 - m^2c^2\,(1+\delta)\,(5+2\delta)\,r_h^2\right) \right]^{1/2} dr. \end{aligned}$$

fnd expand the integrand into the series with respect to the powers of 1/r.
The change in factor in front of 1/r^2 leads to systematic secular precession of the orbit perihelion.

$$\Delta S_r = \Delta S_r^{(0)} - \frac{m^2c^2\,(1+\delta)\,(5+2\delta)\,r_h^2}{2L} \cdot \frac{\partial \Delta S_r^{(0)}}{\partial L}.$$

Then we expand the radial part of the action with respect to the small correction factor in front of 1/r^2 and differentiate this expression with respect to L.

Perihelion precession

$$-\frac{\partial}{\partial L}\Delta S_r^{(0)} = \Delta\varphi^{(0)} = 2\pi.$$

Taking into account these relations we find the result.

Result:

$$\Delta\varphi = \frac{(1+\delta)(2\delta+5)\pi m^2 c^2 r_h^2}{L^2}.$$

Deflection of light

$$g^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k} = 0.$$

- eikonal equation

Result:

$$\Delta\theta = \frac{2(2+\delta)r_h}{\rho}.$$

Shatskiy, A.Reports, 48, 7, (2004)

Comparison with a BH

$$\alpha / r$$

-- classical Newtonian potential;

$$\alpha = (1+\delta) G M m$$

In the classical limit the metrics of a wormhole yields the potential, Which looks like Newtonian;

$$M_0 = (1+\delta) M$$

- measured mass;

Comparison with a BH

$$M_0 = (1+\delta)M$$ - measured mass;

$$\Delta\varphi = \frac{2\delta+5}{1+\delta}\frac{\pi G^2 M_0^2 m^2}{c^2 L^2}$$ - perihelion precession;

$$\Delta\theta = \frac{2+\delta}{1+\delta}\frac{r_g}{\rho},$$ - deflection of light

Comparison with a BH

Perihelion precession

$$\frac{\Delta\varphi}{\Delta\varphi_{BH}} = \frac{2\delta + 5}{6(1+\delta)} = \frac{1}{3} + \frac{1}{2(1+\delta)}.$$

Deflection of light

$$\frac{\Delta\theta}{\Delta\theta_{BH}} = \frac{2 + \delta}{2(1+\delta)} = \frac{1}{2} + \frac{1}{2(1+\delta)}.$$

Выводы:

- Отличие в величине отклонения света для КН и ЧД оказывается порядка малого параметра фантомности.
- Различие между смещениями перигелия для КН и ЧД оказывается существенным (около 15%) даже при ничтожно малой величине фантомности.
- Смещение перигелия в принципе дает еще одну возможность отличить КН от ЧД.
- В Солнце скорее всего нет кротовой норы.